

Turing Machine

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* Introduction :-

[Finite state automata have finite amount of memory & hence cannot do certain things.

eg:-

No finite state automata can accept language $\{a^n b^n \mid n \geq 1\}$. Though it is possible to have finite state automata for adding to arbitrary long binary number we cannot have finite state automata which can multiply to arbitrary long binary number.

To overall ^{come} this problems the hypothetical m/c was propose by ~~the~~ Turing in 1936 & is known as Turing m/c.

[Turing m/c] is a simple mathematical model of the general purpose computer i.e. Turing m/c is capable of performing any calculation which can be perform by any computing m/c.

* "Turing m/c is an ~~atom~~ m/c whose temporary storage is tape. This tape is divided into ~~states~~ ^{cell} each of which is capable of holding one symbol. This ~~stage~~ is infinite tape. The head which is associated with the tape can read or write a symbol & can move left to right or stay in same position". The turing m/c model is given below,

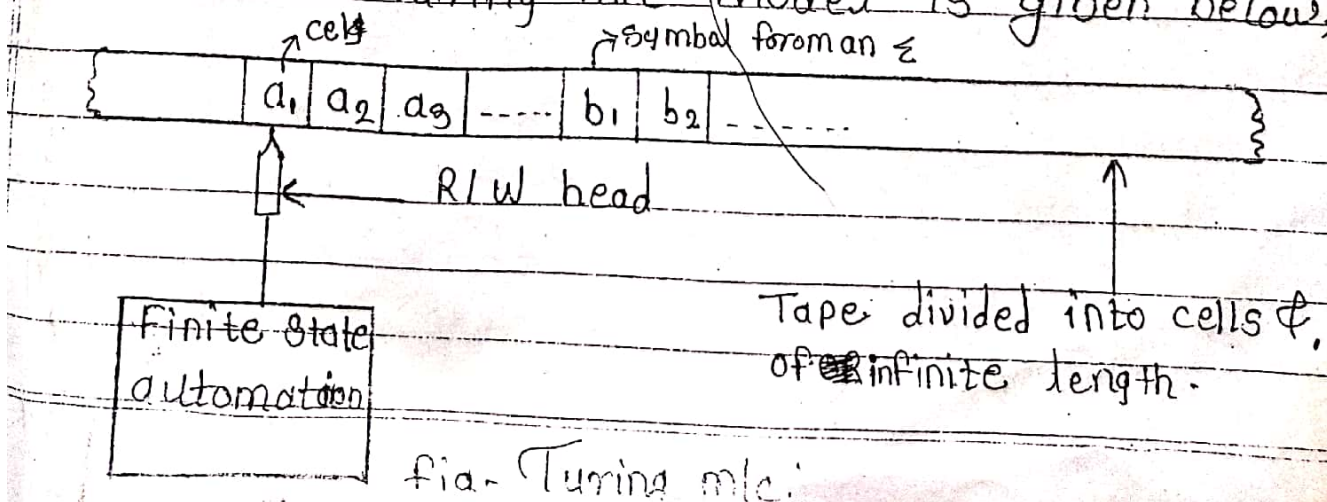


fig. Turing m/c.

In Turing m/c in one move dependent on a symbol scan by the tape (read/write) head & the state of the finite control,

- Changes the state
- prints the symbol on the tape cell scan, replacing what was there
- Moves its head left or right one cell or never in same cell (no move).

✓ * Formal Defⁿ of Turing m/c :-

A Turing m/c 'M' is a 7-tuple as follows
 $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

where,

$Q \rightarrow$ is a finite non-empty set of states

$\Sigma \rightarrow$ is a finite non-empty set of input symbols

$\Sigma \subseteq \Gamma$ and $B \notin \Sigma$.

$\Gamma \rightarrow$ is a finite non-empty set of tape symbols.

$\delta \rightarrow$ is a transition function mapping as,

$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

$q_0 \rightarrow$ is a starting state, $q_0 \in Q$

$B \rightarrow$ is a blank symbol, and $B \in \Gamma$

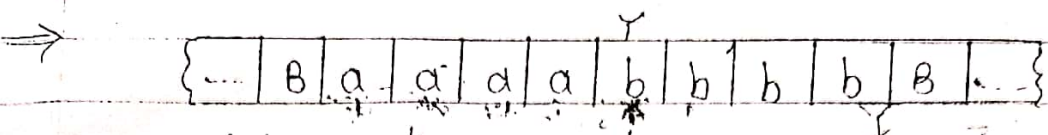
$F \rightarrow$ is a set of final states $\neq F \subseteq Q$.

✓ * Language Acceptability By Turing m/c :-

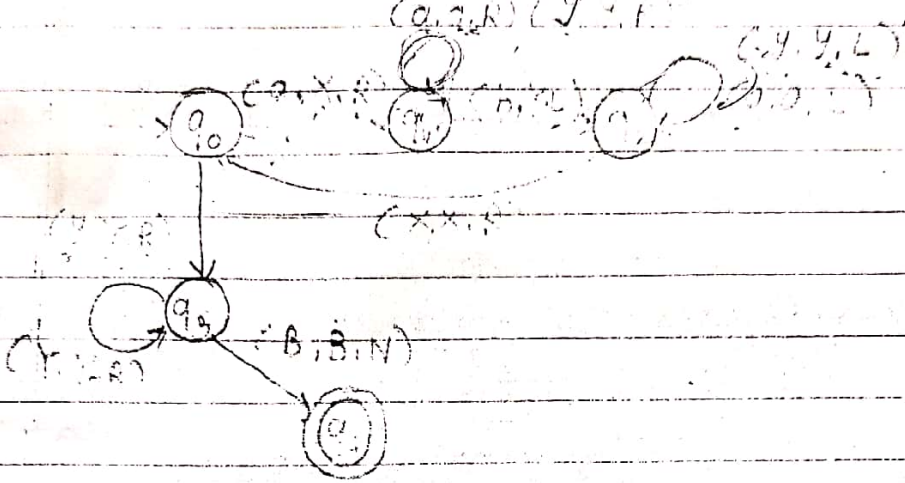
A string 'w' in Σ^* is said to be accepted by Turing m/c 'M' if $q_0 w \vdash^* \alpha_1 p \alpha_2$ for some $p \in F$ & $\alpha_1, \alpha_2 \in \Gamma^*$.

Turing m/c does not accept 'w' if the m/c either halts in a non-accepting state or does not halt.

Q1 Check whether the string is accepted string
 $a^n b^n$
 aaaaabbbb



Transition diagram is,



Transition Table:-

	a	b	X	Y	B
→ q ₀	q ₁ XR	-	-	q ₃ YR	-
q ₁	q ₁ AR	q ₂ YL	-	q ₁ YR	-
q ₂	q ₂ AL	-	q ₀ XR	q ₂ YL	-
q ₃	-	-	-	q ₃ YR	q ₄ BN
⊙ q ₄	acceptance state.				

consider,

- aaaaabbbb ⊢ B q₀ aaaaabbbb B
- ⊢ B X q₁ aaaaabbbb B
- ⊢ B X a q₁ aabbbb B
- ⊢ B X a a q₁ bbbb B
- ⊢ B X a a q₂ a Y bbbb B
- ⊢ B X q₂ a a Y bbb B
- ⊢ B q₂ X a a Y bb B
- ⊢ B X q₀ a a Y bb B
- ⊢ B X X q₁ a Y bb B
- ⊢ B X X a q₁ Y bb B

┌ B X X a Y q₁ b b B

┌ B X X a q₂ Y Y b B

┌ B X X q₂ a Y Y b B

┌ B X q₂ X a Y Y b B

┌ B X X q₀ a Y Y b B

┌ B X X X q₁ Y Y b B

┌ B X X X Y q₁ Y b B

┌ B X X X Y Y q₁ b B

┌ B X X X Y q₂ Y Y B

┌ ~~B X X X Y Y q₂ Y B~~

┌ ~~B X X X Y Y Y q₂ B~~

┌ B X X X q₂ Y Y Y B

┌ B X X q₂ X Y Y Y B

┌ B X X X q₀ Y Y Y B

┌ B X X X Y q₃ Y Y B

┌ B X X X Y Y q₃ Y B

┌ B X X X Y Y Y q₃ B

┌ B X X X Y Y Y q₄ B N

┌ B X X X Y Y Y B N

* Representation Of Turing Machine :-

Turing m/c can be represented with,

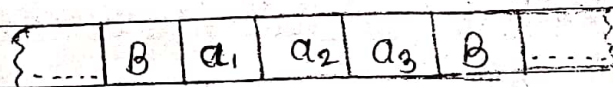
- ① Instantaneous description,
- ② Transition diagram,
- ③ Transition table.

① Instantaneous Description :-

To define it is the current state & i/p string to be processed should be represented with the tape & FSA having read/write head. At the starting the current state is mostly the initial state of Turing m/c.

" An instantaneous description of a turing m/c is a string α, β, δ where ' β ' is a present state of 'M'. The entire i/p string is split as α, δ . The first symbol of ' δ ' is the current symbol 'a' under read/write head. ' δ ' has all the subsequent symbols of the i/p string & the string ' α ' is the substring of the i/p string formed by all the symbol to the left of 'a'.

eg :-



Finite
State
machine

fig - Instantaneous diagram
of turing m/c.

The tape contains the string α, β, δ of the turing m/c, the remaining cell are blanks

ϕ represented by 'B'. The position of the head represents the current state. The 'c' in the above diagram is 'B' & 'd' is $a_2 a_3 \# B$.

② Transition Diagram :-

In transition diagram the states are represented by vertices joint by directed edges.

Transition of state tables with the form (q_i, α, β, d) where, $\alpha, \beta \in \Sigma$, and $d \in \{R, L, N\}$. It means that, $\delta(q_i, \alpha) = (q_j, \beta, d)$

eg :-

see the transition diagram of problem ① means $a^4 b^4$

③ Transition Table :-

The ' δ ' which is use to represent transition function of turing m/c represented in the form of a table is called transition table, if " $\delta(q, a) = (q', \alpha, \beta)$ ", to represent q', α, β of transition in transition diagram under ' q' -column' & ' a -row' where ' q' ' is the resulting state of transition ' a ' is the tape symbol to be return & ' β ' is movement of head.

eg :-

see the transition of the language $L = \{a^4 b^4\}$ problem no. ① //

Unit-5 \rightarrow PDA* Deterministic PDA :-

The PDA is deterministic which has at most one move is possible from any instantaneous description.

Formal defⁿ :-

① A PDA $M = (Q, \Sigma, \Gamma, \delta, z_0, q_0, F)$ is deterministic if, for each q in Q & z_0 in Γ , whenever $\delta(q, \epsilon, z)$ is non empty, then $\delta(q, a, z)$ is empty for all 'a' in Σ .

For no $q, q' \in Q$, z in Γ & 'a' in $\Sigma \cup \{\epsilon\}$ does $\delta(q, a, z)$ contains more than one element.

* Non-deterministic PDA :-

The PDA is non-deterministic which has more than one moves are possible from an instantaneous description.

Formal defⁿ :-

Let, $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ be a PDA. The set NCA) accepted by null store is defined by $NCA) = \{ w \in \Sigma^* / (q_0, w, z_0) \vdash^* (q, \epsilon, \epsilon) \text{ for some } q \in Q \}$.

CHAPTER-2

Imp* Difference :-

NFA

① NFA allows zero or more transition from state to state having i/p symbol from an alphabet and ϵ .

② In NFA they may be zero or more outgoing transition for a symbol from an alphabet $\notin \Sigma$.

③ ' δ ' is a transition function mapping as,

$$\delta: Q \times \Sigma \rightarrow 2^Q$$

④ Every NFA is not always DFA.

⑤ NFA allows ϵ -moves.

DFA

① DFA consist of a finite set of state having transⁿ from state to state that occur on an i/p symbol chosen from an alphabet.

② For each i/p symbol there is exactly one transition out of each state.

③ ' δ ' is a transition function mapping as,

$$\delta: Q \times \Sigma \rightarrow Q$$

④ Every DFA is always NFA.

⑤ DFA has no ϵ -moves.